General Relativity: Problems 1

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1. A μ-meson with an average lifetime of 2×10^{-6} sec is created in the upper atmosphere at an altitude of 6000 m. When it is created it has a velocity of $0.998c$ in a direction towards the earth. What is the average distance that it will travel before decaying, as determined by an observer on earth?

   Consider an observer at rest with respect to the μ-meson. What is the distance he measures from the point of creation of the μ-meson to the earth? Comment on the physical significance of these results.

2. A train 20 metres long tries to get into a garage 10 metres long by driving into it at a speed of $\frac{\sqrt{3}c}{2}$. Show that in the frame of the garage, the whole train can indeed enter the garage before its front strikes the wall. Also calculate the length of the garage as seen by the driver and prove that he expects to strike the wall $\frac{10\sqrt{3}}{c}$ seconds before the back of the train gets in through the back of the garage. Recalling that the maximum speed of propagation of information is c, explain how the train fits into the garage before the news that the front of the train has hit the garage wall reaches the back of the train. Draw a spacetime diagrams from the point of view of the train and the garage to illustrate your answer. Show also that minimum length of the garage for the performance of this trick is $20/\left(\sqrt{3}+2\right)$ metres.

3. A distant camera snaps a photograph of a speeding bullet (velocity v) with length b in its rest frame. Behind the bullet and parallel to its path is a meter stick, at rest with respect to the camera. The direction to the camera is an angle α from the direction of the bullet's velocity. What will be the *apparent length* of the bullet as seen in the photo? (i.e how much of the meter stick is hidden?).

4. Tachyons are hypothetical particles whose velocity is faster than light. Suppose that a tachyon transmitter emits particles of a constant velocity $u > c$ in its rest frame. If a tachyonic message is sent to an observer at rest at a distance L, how much time will elapse before a tachyonic reply can be received? How much time will elapse if the distant observer is moving away at velocity v, and is at a distance L at the instant she receives the message and replies? Show that for $u > c^2[1 + (1 - v^2/c^2)^1/2]/v$ the reply can be received before the signal is sent!